Viet formula. Resolution square trinomials to linear factors

Numbers x_1 and x_2 are solutions of square equation $ax^2+bx+c=0$ if and only if

$$x_1+x_2=-\frac{b}{a} \qquad \text{and} \qquad x_1\cdot x_2=\frac{c}{a}$$

These two equality are called the Viet formula.

What they serve?

The basic application is to help us, when we have solutions x_1 and x_2, to make square

equation: $\quad x^2-(x_1+x_2)x+x_1\cdot x_2=0$

or may be more precise:

$a\left[x^2-(x_1+x_2)x+x_1\cdot x_2\right]=0$, but usually, we take here $a=1$,and the formula is:

Example 1: Write square equation whose solutions are:

a) $x_1=3, x_2=-2$

b) One solution is $x_1=1+2i$

Solution:

a) $x_1=3, x_2=-2$

$$\begin{aligned} x_1+x_2&=3+(-2)=+1 \\ x_1\cdot x_2&=3\cdot(-2)=-6 \end{aligned} \longrightarrow a\left[x^2-(x_1+x_2)x+x_1\cdot x_2\right]=0$$

$$a\left[x^2-\underbrace{(x_1+x_2)}_{1}x+\underbrace{x_1\cdot x_2}_{-6}\right]=0$$

$a\left[x^2-x-6\right]=0$ as we usually take a=1 $\longrightarrow \quad \Rightarrow \; x^2-x-6=0$

b) One solution is $x_1=1+2i$

second solution must be: $x_2=1-2i$

$$x_1+x_2=1+2i+1-2i=2$$
$$x_1\cdot x_2=(1+2i)\cdot(1-2i)=1^2-(2i)^2=1-4i^2=$$

(because $i^2 = -1$) $= 1 + 4 = 5$

$$\underset{2}{\underbrace{x^2 - (x_1 + x_2)}}x + \underset{5}{\underbrace{x_1 \cdot x_2}} = 0 \longrightarrow x^2 - 2x + 5 = 0$$

Example 2. In equation $mx^2 - (3m+1)x + m = 0$ determine the value of the real parameter m

so that is: $x_1 + x_2 = 5$

Solution:

$$\left.\begin{array}{l} a = m \\ b = -(3m+1) \\ c = m \end{array}\right. \longrightarrow \begin{array}{l} x_1 + x_2 = -\dfrac{b}{a} \\ x_1 + x_2 = -\dfrac{-(3m+1)}{m} = \dfrac{3m+1}{m} \end{array}$$

$$x_1 + x_2 = 5 \Rightarrow \quad \begin{array}{l} \dfrac{3m+1}{m} = 5 \\ 3m + 1 = 5m \\ 3m - 5m = -1 \\ -2m = -1 \\ m = \dfrac{1}{2} \end{array}$$

Example 3. In equation $x^2 - 4x + 3(k-1) = 0$ determine the value of the real parameter k,

so that is: $x_1 - 3x_2 = 0$

Solution:

$$\begin{array}{l} a = 1 \\ b = -4 \\ c = 3(k-1) \end{array} \longrightarrow x_1 + x_2 = -\frac{b}{a} = -\frac{-4}{1} = 4$$

$$\left.\begin{array}{l} x_1 + x_2 = 4 \\ x_1 - 3x_2 = 0 \end{array}\right\} \text{ we solve the system}$$

$$\begin{array}{l} x_1 + x_2 = 4 \\ -x_1 + 3x_2 = 0 \end{array}$$

$4x_2 = 4 \Rightarrow x_2 = 1 \Rightarrow x_1 = 3$

How is $x_1 \cdot x_2 = \frac{c}{a} \Rightarrow 3 \cdot 1 = \frac{3(k-1)}{1} \Rightarrow k - 1 = 1 \Rightarrow \boxed{k = 2}$

Example 4. In equation $x^2 - (m+1)x + m = 0$ determine the value of the real parameter m

so that the solutions x_1 and x_2 meet equality $x_1^2 + x_2^2 = 10$

Solution:

$$\begin{aligned} a &= 1 \\ b &= -(m+1) \\ c &= m \end{aligned} \quad \Rightarrow \quad \begin{aligned} x_1 + x_2 &= -\frac{b}{a} = -\frac{-(m+1)}{1} = m+1 \\ x_1 + x_2 &= \frac{c}{a} = \frac{m}{1} = m \end{aligned}$$

This expression $x_1^2 + x_2^2$,often occurs in the tasks.

Move from well-known formula for the square stage: $(x_1 + x_2)^2 = x_1^2 + 2x_1x_2 + x_2^2$

$$\boxed{x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2}$$ **REMEMBER!**

Let's go back to the task:

$$\begin{aligned} x_1^2 + x_2^2 = 10 \Rightarrow (x_1 + x_2)^2 - 2x_1x_2 &= 10 \\ (m+1)^2 - 2m &= 10 \\ m^2 + 2m + 1 - 2m &= 10 \\ m^2 &= 10 - 1 \\ m^2 &= 9 \\ m &= \pm\sqrt{9} \\ m_1 &= 3 \\ m_2 &= -3 \end{aligned}$$

Resolution square trinomials to linear factors

Square trinom by x is a form of expression: ax^2+bx+c ,where $a,b,c \to$ are numbers (coefficients) and $a \neq 0$.

If x_1 and x_2 are solutions of square equation $ax^2+bx+c=0$, then:

$$\boxed{ax^2+bx+c=a(x-x_1)(x-x_2)}$$

Example 1. Square trinom apart on the facts: a) x^2+5x+6
b) x^2+2x+2

Solution:

a) x^2+5x+6

First we solve quadratic equation: $x^2+5x+6=0$

$$\begin{array}{lll} a=1 & D=b^2-4ac & x_{1,2}=\dfrac{-b\pm\sqrt{D}}{2a}=\dfrac{5\pm 1}{2} \\ b=-5 & D=25-24 & x_1=3 \\ c=6 & D=1 & x_2=2 \end{array}$$

$$a(x-x_1)(x-x_2)=1(x-3)(x-2)=(x-3)(x-2)$$

So: $x^2+5x+6=(x-3)(x-2)$

b) x^2+2x+2

$$x^2+2x+2=0 \quad \Rightarrow \quad a=1, b=2, c=2 \qquad D=4-8=-4$$

$$x_{1,2}=\frac{-2\pm 2i}{2}=\frac{2(-1\pm i)}{2}$$

$$x_1=-1+i$$

$$x_2=-1-i$$

$$a(x-x_1)(x-x_2)=1(x+1-i)(x+1+i)$$

So : $x^2+2x+2=(x+1-i)(x+1+i)$

Example 2: Trim fraction: $\dfrac{3x^2+2x-8}{12x^2-7x-12}$

Solution:

I

$$3x^2+2x-8=0$$

$$\begin{array}{lll} a=3 & D=b^2-4ac & x_{1,2}=\dfrac{-b\pm\sqrt{D}}{2a} \\ b=2 & D=4+4\cdot 3\cdot 8 & x_{1,2}=\dfrac{-2\pm 10}{6} \\ c=-8 & D=4+36 & x_1=\dfrac{-2+10}{6}=\dfrac{8}{6}=\dfrac{4}{3} \\ & D=100 & x_2=\dfrac{-2-10}{6}=-2 \end{array}$$

$$3x^2+2x-8=a(x-x_1)(x-x_2)=3(x-\frac{4}{3})(x+2)$$

II

$$12x^2-7x-12=0$$

$$\begin{array}{lll} a=12 & D=b^2-4ac & x_{1,2}=\dfrac{-b\pm\sqrt{D}}{2a} \\ b=-7 & D=4+4\cdot 3\cdot(-12) & x_{1,2}=\dfrac{-7\pm 25}{24} \\ c=-12 & D=49+576 & x_1=\dfrac{7+25}{24}=\dfrac{32}{24}=\dfrac{4}{3} \\ & D=625 & x_2=\dfrac{7-25}{24}=\dfrac{18}{24}=-\dfrac{3}{4} \end{array}$$

$$12x^2-7x-12=a(x-x_1)(x-x_2)=12(x-\frac{4}{3})(x+\frac{3}{4})$$

Let's go back now in the fraction:

$$\frac{3x^2+2x-8}{12x^2-7x-12}=\frac{3(x-\frac{4}{3})(x+2)}{12(x-\frac{4}{3})(x+\frac{3}{4})}=\frac{x+2}{4(x+\frac{3}{4})}$$

Of course, with the conditions: $\begin{array}{l} x-\dfrac{4}{3}\neq 0 \\ x\neq\dfrac{4}{3} \end{array}$ and $\begin{array}{l} x+\dfrac{3}{4}\neq 0 \\ x\neq -\dfrac{3}{4} \end{array}$

Example 3. Trim fraction: $\frac{x^3+1}{x^2-2x-3}$

Solution:

I

$$x^2-2x-3=0$$

$$a=1 \qquad D=b^2-4ac \qquad x_{1,2}=\frac{-b\pm\sqrt{D}}{2a}$$
$$b=-2 \qquad D=4+12 \qquad x_{1,2}=\frac{2\pm 4}{2}$$
$$c=-3 \qquad D=16 \qquad x_1=3$$
$$x_2=-1$$

$$x^2-2x-3=a(x+x_1)(x+x_2)=1(x-3)(x-(-1))=(x-3)(x+1)$$

II

for $x^3+1\rightarrow$ we have formula: $A^3+B^3=(A+B)(A^2-AB+B^2)$

So : $x^3+1=(x+1)(x^2-x+1)$

Let's go back to the fraction:

$$\frac{x^3+1}{x^2-2x-3}=\frac{(x+1)(x^2-x+1)}{(x-3)(x+1)}=\frac{x^2-x+1}{x-3}$$

of course with the condition : $\begin{matrix} x-3\neq 0 \\ x\neq 3\end{matrix}$ and $\begin{matrix} x+1\neq 0 \\ x\neq -1\end{matrix}$