

## Viet formula. Resolution square trinomials to linear factors

Numbers  $x_1$  and  $x_2$  are solutions of square equation  $ax^2 + bx + c = 0$  if and only if

$$x_1 + x_2 = -\frac{b}{a} \quad \text{and} \quad x_1 \cdot x_2 = \frac{c}{a}$$

These two equality are called the Viet formula.

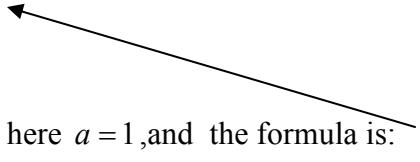
What they serve?

The basic application is to help us, when we have solutions  $x_1$  and  $x_2$ , to make square

equation: 
$$x^2 - (x_1 + x_2)x + x_1 \cdot x_2 = 0$$

or may be more precise:

$a[x^2 - (x_1 + x_2)x + x_1 \cdot x_2] = 0$ , but usually, we take here  $a = 1$ , and the formula is:



**Example 1:** Write square equation whose solutions are:

a)  $x_1 = 3, x_2 = -2$

b) One solution is  $x_1 = 1 + 2i$

**Solution:**

a)  $x_1 = 3, x_2 = -2$

$$\begin{aligned} x_1 + x_2 &= 3 + (-2) = +1 \\ x_1 \cdot x_2 &= 3 \cdot (-2) = -6 \end{aligned} \longrightarrow a[x^2 - (x_1 + x_2)x + x_1 \cdot x_2] = 0$$

$$a \left[ x^2 - \underbrace{(x_1 + x_2)}_{1} x + \underbrace{x_1 \cdot x_2}_{-6} \right] = 0$$

$$a[x^2 - x - 6] = 0 \quad \text{as we usually take } a=1 \longrightarrow \Rightarrow x^2 - x - 6 = 0$$

b) One solution is  $x_1 = 1 + 2i$

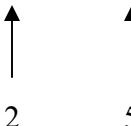
second solution must be:  $x_2 = 1 - 2i$

$$x_1 + x_2 = 1 + 2i + 1 - 2i = 2$$

$$x_1 \cdot x_2 = (1 + 2i) \cdot (1 - 2i) = 1^2 - (2i)^2 = 1 - 4i^2 =$$

(because  $i^2 = -1$ ) = 1 + 4 = 5

$$x^2 - (x_1 + x_2)x + x_1 \cdot x_2 = 0 \quad \xrightarrow{\quad} \quad x^2 - 2x + 5 = 0$$


  
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**Example 2.** In equation  $mx^2 - (3m+1)x + m = 0$  determine the value of the real parameter  $m$

so that is:  $x_1 + x_2 = 5$

**Solution:**

$$\begin{array}{l} a = m \\ b = -(3m+1) \\ c = m \end{array} \quad \xrightarrow{\quad} \quad \begin{array}{l} x_1 + x_2 = -\frac{b}{a} \\ x_1 + x_2 = -\frac{-(3m+1)}{m} = \frac{3m+1}{m} \end{array}$$

$$\begin{aligned} x_1 + x_2 = 5 &\Rightarrow \frac{3m+1}{m} = 5 \\ 3m+1 &= 5m \\ 3m - 5m &= -1 \\ -2m &= -1 \\ m &= \frac{1}{2} \end{aligned}$$

**Example 3.** In equation  $x^2 - 4x + 3(k-1) = 0$  determine the value of the real parameter  $k$ ,

so that is:  $x_1 - 3x_2 = 0$

**Solution:**

$$\begin{array}{l} a = 1 \\ b = -4 \\ c = 3(k-1) \end{array} \quad \xrightarrow{\quad} \quad x_1 + x_2 = -\frac{b}{a} = -\frac{-4}{1} = 4$$

$$\left. \begin{array}{l} x_1 + x_2 = 4 \\ x_1 - 3x_2 = 0 \end{array} \right\} \text{we solve the system}$$


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$$\begin{array}{r} x_1 + x_2 = 4 \\ -x_1 + 3x_2 = 0 \end{array} \quad \underline{\underline{\quad}}$$

$$4x_2 = 4 \Rightarrow x_2 = 1 \Rightarrow x_1 = 3$$

How is  $x_1 \cdot x_2 = \frac{c}{a} \Rightarrow 3 \cdot 1 = \frac{3(k-1)}{1} \Rightarrow k-1=1 \Rightarrow \boxed{k=2}$

**Example 4.** In equation  $x^2 - (m+1)x + m = 0$  determine the value of the real parameter  $m$

so that the solutions  $x_1$  and  $x_2$  meet equality  $x_1^2 + x_2^2 = 10$

**Solution:**

$$\begin{array}{l} a=1 \\ b=-(m+1) \\ c=m \end{array} \Rightarrow \begin{array}{l} x_1 + x_2 = -\frac{b}{a} = -\frac{-(m+1)}{1} = m+1 \\ x_1 \cdot x_2 = \frac{c}{a} = \frac{m}{1} = m \end{array}$$

This expression  $x_1^2 + x_2^2$ , often occurs in the tasks.

Move from well-known formula for the square stage:  $(x_1 + x_2)^2 = x_1^2 + 2x_1x_2 + x_2^2$

$$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2$$

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**REMEMBER!**

Let's go back to the task:

$$\begin{aligned} x_1^2 + x_2^2 &= 10 \Rightarrow (x_1 + x_2)^2 - 2x_1x_2 = 10 \\ (m+1)^2 - 2m &= 10 \\ m^2 + 2m + 1 - 2m &= 10 \\ m^2 &= 10 - 1 \\ m^2 &= 9 \\ m &= \pm\sqrt{9} \\ m_1 &= 3 \\ m_2 &= -3 \end{aligned}$$

## **Resolution square trinomials to linear factors**

Square trinom by x is a form of expression:  $ax^2 + bx + c$ , where  $a, b, c \rightarrow$  are numbers (coefficients) and  $a \neq 0$ .

If  $x_1$  and  $x_2$  are solutions of square equation  $ax^2 + bx + c = 0$ , then:

$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

Example 1. Square trinom apart on the facts:

### Solution:

a)  $x^2 + 5x + 6$

First we solve quadratic equation:  $x^2 + 5x + 6 = 0$

$$\begin{array}{lll} a=1 & D=b^2-4ac & x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{5 \pm 1}{2} \\ b=-5 & D=25-24 & x_1 = 3 \\ c=6 & D=1 & x_2 = 2 \end{array}$$

$$a(x - x_1)(x - x_2) = 1(x - 3)(x - 2) = (x - 3)(x - 2)$$

$$\text{So: } x^2 + 5x + 6 = (x - 3)(x - 2)$$

b)  $x^2 + 2x + 2$

$$x^2 + 2x + 2 = 0 \Rightarrow a=1, b=2, c=2 \quad D=4-8=-4$$

$$x_{1,2} = \frac{-2 \pm 2i}{2} = \frac{2(-1 \pm i)}{2}$$

$$x_1 = -1 + i$$

$$x_2 = -1 - i$$

$$a(x - x_1)(x - x_2) = 1(x + 1 - i)(x + 1 + i)$$

$$\text{So } : x^2 + 2x + 2 = (x + 1 - i)(x + 1 + i)$$

**Example 2:** Trim fraction:  $\frac{3x^2 + 2x - 8}{12x^2 - 7x - 12}$

Solution:

I

$$3x^2 + 2x - 8 = 0$$

$$\begin{aligned} a &= 3 & D &= b^2 - 4ac & x_{1,2} &= \frac{-b \pm \sqrt{D}}{2a} \\ b &= 2 & D &= 4 + 4 \cdot 3 \cdot 8 & x_{1,2} &= \frac{-2 \pm 10}{6} \\ c &= -8 & D &= 4 + 36 & x_1 &= \frac{-2 + 10}{6} = \frac{8}{6} = \frac{4}{3} \\ & & D &= 100 & x_2 &= \frac{-2 - 10}{6} = -2 \end{aligned}$$

$$3x^2 + 2x - 8 = a(x - x_1)(x - x_2) = 3\left(x - \frac{4}{3}\right)(x + 2)$$

II

$$\begin{aligned} 12x^2 - 7x - 12 &= 0 & x_{1,2} &= \frac{-b \pm \sqrt{D}}{2a} \\ a &= 12 & D &= b^2 - 4ac & x_{1,2} &= \frac{-7 \pm 25}{24} \\ b &= -7 & D &= 4 + 4 \cdot 3 \cdot (-12) & x_1 &= \frac{7 + 25}{24} = \frac{32}{24} = \frac{4}{3} \\ c &= -12 & D &= 49 + 576 & x_2 &= \frac{7 - 25}{24} = \frac{18}{24} = -\frac{3}{4} \\ & & D &= 625 & & \end{aligned}$$

$$12x^2 - 7x - 12 = a(x - x_1)(x - x_2) = 12\left(x - \frac{4}{3}\right)\left(x + \frac{3}{4}\right)$$

Let's go back now in the fraction:

$$\frac{3x^2 + 2x - 8}{12x^2 - 7x - 12} = \frac{3\left(x - \frac{4}{3}\right)\left(x + 2\right)}{12\left(x - \frac{4}{3}\right)\left(x + \frac{3}{4}\right)} = \frac{x + 2}{4\left(x + \frac{3}{4}\right)}$$

$$\begin{aligned} x - \frac{4}{3} &\neq 0 & x + \frac{3}{4} &\neq 0 \\ x &\neq \frac{4}{3} & x &\neq -\frac{3}{4} \end{aligned}$$

Of course, with the conditions:

**Example 3.** Trim fraction:  $\frac{x^3 + 1}{x^2 - 2x - 3}$

Solution:

$$\text{I} \\ x^2 - 2x - 3 = 0$$

$$\begin{array}{lll} a = 1 & D = b^2 - 4ac & x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} \\ b = -2 & D = 4 + 12 & x_{1,2} = \frac{2 \pm 4}{2} \\ c = -3 & D = 16 & x_1 = 3 \\ & & x_2 = -1 \end{array}$$

$$x^2 - 2x - 3 = a(x + x_1)(x + x_2) = 1(x - 3)(x - (-1)) = (x - 3)(x + 1)$$

**II**

for  $x^3 + 1 \rightarrow$  we have formula:  $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$

So :  $x^3 + 1 = (x + 1)(x^2 - x + 1)$

Let's go back to the fraction:

$$\frac{x^3 + 1}{x^2 - 2x - 3} = \frac{(x + 1)(x^2 - x + 1)}{(x - 3)(x + 1)} = \frac{x^2 - x + 1}{x - 3}$$

of course with the condition :  $\begin{array}{l} x - 3 \neq 0 \\ x \neq 3 \end{array}$  and  $\begin{array}{l} x + 1 \neq 0 \\ x \neq -1 \end{array}$